International Journal of Theoretical Physics, Vol. 29, No. 6, 1990

Gravitational Wave Due to Explosion of SN 1987A

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Received September 15, 1989

Observations of the gravitational waves from the explosion of SN 1987A are shown to be more consistent with a new special relativistic theory of gravitation than with predictions of general relativity.

The explosion of SN 1987A affords a rare chance to observe the gravitational field due to the action of extradense material. The observation proved that there is a gravitational wave due to the explosion of SN 1987A. But this has not been accepted extensively, because the strength of the gravitational wave determined by the experiment is 1000 times the value predicted by theory. Zhang Junhao and Chen Xiang (1990) showed how one can set up a new gravitational theory based on special relativity. The purpose of this paper is to show that the observational result of the gravitational wave of SN 1987A is reasonable from the viewpoint of the new gravitational theory.

From the new gravitational theory, the field equation is

$$\frac{\partial^2 A_{\mu\nu}}{\partial x_\alpha\, \partial x_\alpha}+\lambda\frac{\partial A_{\mu\omega}\, \partial A_{\nu\omega}}{\partial x_\alpha\, \partial x_\alpha}-\frac{1}{2}\delta_{\mu\nu}\left(\frac{\partial^2 A_{\tau\tau}}{\partial x_\alpha\, \partial x_\alpha}+\lambda\frac{\partial A_{\tau\omega}\, \partial A_{\tau\omega}}{\partial x_\alpha\, \partial x_\alpha}\right)=-\frac{8\pi G\rho}{c^4}U_\mu U_\nu \tag{1}$$

where $A_{\mu\nu}$ is a symmetric tensor, and ρ is the density in a static frame of reference. The preceding work pointed out that $\lambda=6\Delta$, where Δ is the relative error of the perihelion shift. It is defined as $\Delta=(\Delta\varphi-\Delta\varphi_0)/\Delta\varphi_0$, where $\Delta\varphi$ is the experimental value of the perihelion shift, and $\Delta\varphi_0$ is the expected value from general relativity. The present experiment allows $\Delta_{\max}<0.1$; therefore $\lambda_{\max}<0.6$.

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0020-7748/90/0600-0599$06.00/0

The first step is to simplify the problem. Suppose that the supernova kept its spherical symmetry in the contractive process. Its radius may be expressed by

$$R(t)=R_0-vt \tag{2}$$

where R_0 is the initial radius, and v is the velocity of contraction. In this case, we can prove that the gravitational field has spherical symmetry. This means that $A_{\mu\nu}$ has the form

$$\begin{aligned} A_{44}(\mathbf{r},t)&=\xi(r,t)\\ A_{4k}(\mathbf{r},t)&=A_{k4}(\mathbf{r},t)=\eta(r,t)x_k/r\\ A_{kl}(\mathbf{r},t)&=\zeta(r,t)\delta_{kl}+\theta(r,t)x_kx_l/r^2 \end{aligned} \tag{3}$$

Substituting (3) into (1), we get

$$\begin{aligned} &\left(\frac{\partial^2\xi}{\partial r^2}+\frac{2}{r}\frac{\partial\xi}{\partial r}-\frac{\partial^2\xi}{c^2\partial t^2}\right)\\ &\qquad+\lambda\left[\left(\frac{\partial\xi}{\partial r}\right)^2-\frac{1}{c^2}\left(\frac{\partial\xi}{\partial t}\right)^2+\left(\frac{\partial\eta}{\partial r}\right)^2+\frac{2\eta^2}{r^2}-\frac{1}{c^2}\left(\frac{\partial\eta}{\partial t}\right)^2\right]\\ &=\frac{4\pi G\rho(r,t)(1+\beta^2)}{c^2(1-\beta^2)}\\ &\left(\frac{\partial^2\eta}{\partial r^2}+\frac{2}{r}\frac{\partial\eta}{\partial r}-\frac{2\eta}{r^2}-\frac{\partial^2\eta}{c^2\partial t^2}\right)\\ &\qquad+\left[\frac{\partial\eta}{\partial r}\left(\frac{\partial\xi}{\partial r}+\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial r}\right)-\frac{\partial\eta}{c^2\partial t}\left(\frac{\partial\xi}{\partial t}+\frac{\partial\zeta}{\partial t}+\frac{\partial\theta}{\partial t}\right)\right]\\ &=\frac{-8\pi iG\rho(r,t)\beta}{c^2(1-\beta^2)}\\ &\left(\frac{\partial^2\theta}{\partial r^2}+\frac{2}{r}\frac{\partial\theta}{\partial r}-\frac{6\theta}{r^2}-\frac{\partial^2\theta}{c^2\partial t^2}\right)\\ &\qquad+\lambda\left\{2\frac{\partial\theta}{\partial r}\frac{\partial\zeta}{\partial r}+\left(\frac{\partial\theta}{\partial r}\right)^2+\left(\frac{\partial\eta}{\partial r}\right)^2+\frac{\theta^2-\eta^2}{r^2}\right.\\ &\qquad\left.-\frac{1}{c^2}\left[\left(\frac{\partial\theta}{\partial t}\right)^2+2\frac{\partial\theta}{\partial t}\frac{\partial\zeta}{\partial t}+\left(\frac{\partial\eta}{\partial t}\right)^2\right]\right\}\\ &=-\frac{8\pi G\rho(r,t)\beta^2}{c^2(1-\beta^2)} \end{aligned} \tag{4}$$

$$\left(\frac{\partial^2\zeta}{\partial r^2}+\frac{2}{r}\frac{\partial\zeta}{\partial r}+\frac{2\theta}{r^2}-\frac{\partial^2\zeta}{c^2\,\partial t^2}\right)$$
$$+\lambda\left[\left(\frac{\partial\zeta}{\partial r}\right)^2+\frac{\theta^2+\eta^2}{r^2}-\frac{1}{c^2}\left(\frac{\partial\zeta}{\partial t}\right)^2\right]$$
$$=\frac{-4\pi G\rho(r,t)}{c^2}$$

where $\beta = v/c$.

Next, we use the fact that β is small. Suppose that each function in (4) can be expanded in a β-power series, for example.

$$\zeta(r,t)=\zeta^{(0)}(r,t)+\beta\zeta^{(1)}(r,t)+\cdots+\beta^N\zeta^{(N)}(r,t)+\cdots \tag{5}$$

Inserting the expansion of each function into (4), we can obtain a set of approach equations, and get solutions

$$\theta^{(0)}(r,t)=\theta^{(1)}(r,t)=\eta^{(0)}(r,t)=0 \tag{6}$$

Further, if we take only the zeroth- and first-order approximations into account, we have

$$\left(\frac{\partial^2\xi}{\partial r^2}+\frac{2}{r}\frac{\partial\xi}{\partial r}-\frac{\partial^2\xi}{c^2\,\partial t^2}\right)+\lambda\left(\frac{\partial\xi}{\partial r}\frac{\partial\xi}{\partial r}-\frac{\partial\xi}{c^2\,\partial t}\frac{\partial\xi}{\partial t}\right)$$
$$=\frac{4\pi G\rho(r,t)}{c^2} \tag{7}$$

$$\left(\frac{\partial^2\zeta}{\partial r^2}+\frac{2}{r}\frac{\partial\zeta}{\partial r}-\frac{\partial^2\zeta}{c^2\,\partial t^2}\right)+\lambda\left(\frac{\partial\zeta}{\partial r}\frac{\partial\zeta}{\partial r}-\frac{\partial\zeta}{c^2\,\partial t}\frac{\partial\zeta}{\partial t}\right)$$
$$=\frac{-4\pi G\rho(r,t)}{c^2} \tag{8}$$

and

$$\left(\frac{\partial^2\eta^{(1)}}{\partial r^2}+\frac{2}{r}\frac{\partial\eta^{(1)}}{\partial r}-\frac{2\eta^{(1)}}{r^2}-\frac{\partial^2\eta^{(1)}}{c^2\,\partial t^2}\right)$$
$$+\lambda\left[\frac{\partial\eta^{(1)}}{\partial r}\left(\frac{\partial\xi^{(0)}}{\partial r}+\frac{\partial\zeta^{(0)}}{\partial r}\right)\right.$$
$$\left.-\frac{\partial\eta^{(1)}}{c^2\,\partial t}\left(\frac{\partial\xi^{(0)}}{\partial t}+\frac{\partial\zeta^{(0)}}{\partial t}\right)\right]$$
$$=\frac{-8i\pi G\rho(r,t)}{c^2} \tag{9}$$

This set of equations is simpler than the original equations. The distinction between (7) and (8) is only the sign on the right side. This characteristic is important for the further discussion.

Suppose that

$$\zeta(r, t) = \lambda^{-1} \ln[Y(r, t)/r] \tag{10}$$

then (7) becomes

$$\frac{\partial^2 Y(r, t)}{\partial r^2} - \frac{\partial^2 Y(r, t)}{c^2 \partial t^2} = \frac{-4\pi\lambda G\rho(r, t)}{c^2} Y(r, t) \tag{11}$$

Obviously, if one changes the sign of the right side of (11), then (10) is the expression for ξ.

Let us introduce a new variable

$$\tau = 1/R(t) \tag{12}$$

Then, inside of the star, we have

$$\frac{\partial^2 Y^{(i)}(r, \tau)}{\partial r^2} - \beta^2\left(\tau^4 \frac{\partial^2}{\partial \tau^2} + 2\tau^3 \frac{\partial}{\partial \tau}\right) Y^{(i)}(r, \tau) = -\omega\tau^3 Y^{(i)}(r, \tau) \tag{13}$$

where

$$\omega^2 = 3\lambda GM/c^2 \tag{14}$$

Suppose that

$$Y^{(i)}(r, \tau) = Y_0^{(i)}(r, \tau) + \beta^2 Y_2^{(i)}(r, \tau) + \beta^4 Y_4^{(i)}(r, \tau) + \cdots \tag{15}$$

Then (13) becomes a set of approximate equations

$$\partial^2 Y_0^{(i)}(r, \tau)/\partial r^2 = -\omega^2\tau^3 Y_0^{(i)}(r, \tau) \tag{16}$$

$$\partial^2 Y_{2n+2}^{(i)}(r, \tau)/\partial r^2 = -\omega^2\tau^3 Y_{2n+2}^{(i)}(r, \tau) + \left(\tau^4 \frac{\partial^2}{\partial \tau^2} + 2\tau^3 \frac{\partial}{\partial \tau}\right) Y_{2n}^{(i)}(r, \tau) \tag{17}$$

The solution of the approximate equation (16) is

$$Y_0^{(i)}(r, \tau) = A_0(\tau) \sin(\omega\tau^{3/2} r) \tag{18}$$

where we have used the condition that the limit of $\zeta(r, \tau)$ is finite if $r \to 0$. Here $A_0(\tau)$ is an arbitrary function of τ. As mentioned above, equation (8) [or equation (13)] is a zeroth- and first-order approximate equation; therefore, we take only the same approximation of $Y^{(i)}(r, \tau)$ into account.

In the region outside the star, (11) becomes

$$\frac{\partial^2 Y^{(e)}(r, t)}{\partial r^2} - \frac{\partial^2 Y^{(e)}(r, t)}{c^2 \partial t^2} = 0 \tag{19}$$

$$Y^{(e)}(r, t) = r + f(r - ct) \tag{20}$$

where f is an arbitrary function of $(r-ct)$. The $Y^{(i)}(r, t)$ and $Y^{(e)}(r, t)$ satisfy the boundary conditions

$$Y_0^{(i)}(r, t)|_{r=R(t)} = Y^{(e)}(r, t)|_{r=R(t)} \quad (21)$$

$$\partial Y_0^{(i)}(r, t)/\partial r|_{r=R(t)} = \partial Y^{(e)}(r, t)/\partial r|_{r=R(t)} \quad (22)$$

From these equations, we obtain

$$f(r-ct_1) = \frac{3\lambda GM}{c^2\alpha^2}\left(-1+\frac{1}{\alpha}\tan\alpha+\frac{3\beta}{2\alpha^2}\tan^2\alpha-\frac{\beta}{2\alpha}\tan^3\alpha\right) \quad (23)$$

$$\lim_{r\to\infty}\zeta^{(e)}(r-ct_1) = \frac{3GM}{c^2r\alpha^2}\left(-1+\frac{1}{\alpha}\tan\alpha+\frac{3\beta}{2\alpha^2}\tan^2\alpha-\frac{\beta}{2\alpha}\tan^3\alpha\right) \quad (24)$$

where

$$\alpha = \{3\lambda GM/[c^2(R_0-vt)]\}^{1/2} \quad (25)$$

Suppose that r and t_1 are the distance from the star and the time of observation. Then we have the relation

$$c(t_1-t) = r-R(t) = r-(R_0-vt) \quad (26)$$

and the expression for α may be rewritten

$$\alpha(r-ct_1) = \left\{\frac{3\lambda GM(1+\beta)}{c^2[\beta(r-ct_1)+R_0]}\right\}^{1/2} \quad (27)$$

Note that equation (24) contains the tangential function. If a celestial body is of medium density, such as the sun, then $\alpha < 0.15$, and the expansion,

$$\tan\alpha = \alpha+\tfrac{1}{3}\alpha^3+\tfrac{2}{15}\alpha^5+\cdots$$

is enough to take account of the contribution of the nonlinear term in (1). It is easy to prove that the correction due to the nonlinear term is very weak. For an extradense body, such as a supernova, the result is totally different. The mass of a supernova is roughly $M \approx 2.8\times10^{30}$ kg. The minimal radius is approximate 800 m. While we do not yet know the exact value of λ from the experiment, $\lambda_{\max} < 0.6$ from the perihelion shift, and therefore we can obtain $\alpha_{\max} \sim 2 > \pi/2$. If α tends to $\pi/2$, then (24) becomes

$$\lim_{\alpha\to\pi/2}\zeta^{(e)}(r-ct_1) \approx \frac{3GM\tan\alpha}{c^2r\alpha^3}\left(1+\frac{3\beta}{2\alpha}\tan\alpha-\frac{\beta}{2}\tan^2\alpha\right) \quad (28)$$

It will exceed enormously the expected value of the other theory. Then $\alpha_c = \pi/2$ may be called the critical angle, and the corresponding radius

$$R_c = 12\lambda GM/(c\pi)^2 \quad (29)$$

may be called the critical radius.

As mentioned above, if one changes the sign of the right side of (13), then $\zeta^{(e)}(r, t)$ changes into $\xi^{(e)}(r, t)$,

$$\lim_{r\to\infty} \xi^{(e)}(r, t_1) = \frac{3GM}{c^2 r\alpha^2}\left(-1 + \frac{1}{\alpha}\,\text{th}\,\alpha + \frac{3\beta}{2\alpha^2}\,\text{th}^2\,\alpha + \frac{\beta}{2\alpha}\,\text{th}^3\,\alpha\right) \tag{30}$$

The distinction of the sign of the field source term between (7) and (8) is important. Because of this, if the expression for $\zeta^{(e)}(r, t)$ contains $\tan\alpha$, then the expression for $\xi^{(e)}(r, t)$ contains $\text{th}\,\alpha$.

In the same manner, we obtain

$$\lim_{r\to\infty} \eta^{(e)}(r, t) = 3i\beta GM/(4c^2 r) \tag{31}$$

Zhang Junhao and Chen Xiang (1990) showed that, in the reference frame where the center of mass of the supernova is static, the gravitational force acting on a body is

$$\mathbf{F} = m\left\{\left[-c^2\left(\frac{\partial\xi}{\partial r} - \frac{\partial\eta}{ic\,\partial t}\right) + ic\left(\frac{\partial\eta}{\partial r} - \frac{\eta}{r} - \frac{\partial\theta}{ic\,\partial t}\right)u_r + \left(\frac{\partial\zeta}{\partial r} - \frac{\theta}{r}\right)u^2\right]\mathbf{r}^0 + \left[ic\left(\frac{\eta}{r} - \frac{\partial\zeta}{ic\,\partial t}\right) - \left(\frac{\partial\zeta}{\partial r} - \frac{\theta}{r}\right)u_r\right]\mathbf{u}\right\} \tag{32}$$

where m and $\mathbf{u}$ are the mass and velocity of the body, and $\mathbf{r}^0$ is the unit vector of the radial direction. From (28), we have

$$\frac{\partial\zeta^{(e)}}{\partial r} = -\frac{3GM\tan\alpha}{c^2 r^2\alpha^3}\left(1 + \frac{3\beta}{2\alpha}\tan\alpha - \frac{\beta}{2}\tan^2\alpha\right) + \frac{3GM}{c^2 r}\frac{\partial\alpha}{\partial r}\frac{d}{d\alpha}\left[\frac{\tan\alpha}{\alpha^3}\left(1 + \frac{3\beta}{2\alpha}\tan\alpha - \frac{\beta}{2}\tan^2\alpha\right)\right] \tag{33}$$

where

$$\frac{\partial\alpha}{\partial r} = -\frac{\partial\alpha}{c\,\partial t} = -\frac{\beta\alpha}{2(1+\beta)R(t)} \tag{34}$$

Because $r \gg R(t)$, the contribution of the second term of (33) is greater than that of the first term. So we have

$$\frac{\partial\zeta^{(e)}}{\partial r} \approx -\frac{\partial\zeta^{(e)}}{c\,\partial t} \approx \frac{-3GM}{2c^2 rR}\,\frac{\beta}{1+\beta}\,\frac{1}{\alpha^2\cos^2\alpha}\left(1 - \frac{3\beta}{2}\tan^2\alpha\right) \tag{35}$$

$$\frac{\partial\xi^{(e)}}{\partial r} \approx \frac{-3GM}{2c^2 rR}\,\frac{\beta}{1+\beta}\,\frac{1}{\alpha^2}\left(2 + \frac{1}{\text{ch}^2\alpha} - \frac{3}{\alpha}\,\text{th}\,\alpha\right) \tag{36}$$

Further, we have

$$\mathbf{F} \approx m\left[-c^2\frac{\partial \xi^{(e)}}{\partial r}\mathbf{r}^0 + \frac{\partial \zeta^{(e)}}{\partial r}c\mathbf{u}\right]$$

$$\approx \frac{3GMm^\beta}{2rR\alpha^2}\left[\left(2+\frac{1}{\mathrm{ch}^2\,\alpha}-\frac{3}{\alpha}\,\mathrm{th}\,\alpha\right)\mathbf{r}^0 - \frac{1}{\cos^2\alpha}\left(1-\frac{3\beta}{2}\tan^2\alpha\right)\frac{\mathbf{u}}{c}\right] \tag{37}$$

where α is determined by λ, M, and R. Since these values have not yet been measured exactly, we do not know the exact value of α or $(\cos\alpha)^{-1}$. But, as has been pointed out, it is possible that α tends to $\alpha_c = \pi/2$ for the supernova explosion. Therefore, from the new gravitational theory, the strength of the gravitational wave of SN 1987A (Geneva) is reasonable and permissible. Conversely, we can deem that α tends to α_c for the explosion of SN 1987A, and determine further λ or R from α_c.

However, in the different gravitational theories, all of the field equations are nonlinear, but they take different forms. The distinction between the results of different nonlinear equations is small if the gravitational field is due to medium density material, but is obvious for extradense material. Thus, the explosion of a supernova provides a rare chance to test different gravitational theories. The observation of SN 1987A favors our gravitational theory.

REFERENCES

Zhang Junhao and Chen Xiang (1990). Special relativistic gravitational theory, *International Journal of Theoretical Physics*, this issue.